

In the Specification:

Please replace the following paragraphs:

a1 [0020] Like most of the ABS control algorithm, the current controller also requires the knowledge of wheel slip. The objective of the controller is to keep the wheel slip at a value that would maximize the tire-road adhesion (or minimize the tire slip). This is unlike previously known systems that oscillate greatly and have greater variations in slip angles. Normalized tire slip is obtained from the following definition:

$$\kappa_i(t) = \frac{V - R\omega_i}{V}$$

$$\kappa_i(t) = \frac{V - R\omega_i}{V} \quad (1)$$

where

R = Effective rolling radius for the tire

$\omega_i$  = Wheel rotational speed for i-th tire

V = Vehicle longitudinal speed in road co-ordinate system.

a2 [0021] It is necessary to obtain the dynamic equations for the vehicle motion in order to develop the control algorithm. A simplified vehicle model is obtained for a straight line braking event. The vehicle motion in the longitudinal direction on the road plane is described by the following equation.

$$\sum F_{xr} = F_{xsumr} + F_{txr} - F_{axr} = M(\dot{V} - V_y r_r) + m_s \dot{Z}_{sr}$$

$$\sum F_{xr} = F_{xsumr} + F_{txr} - F_{axr} = M(\dot{V} - V_y r_r) + m_s \dot{Z}_{sr} q_r$$

where

$F_{xsumr}$  = sum of road forces in the x-direction at the road tire interfaces

$F_{txr}$  = Terrain forces at the c.g. arising out of road slopes and grades

$F_{axr}$  = Aerodynamic drag forces on the vehicle

M = Total vehicle mass

$\dot{V}$  = Vehicle longitudinal velocity

$V_y$  = Vehicle lateral velocity

$r_r$  = Vehicle yaw velocity

$m_s$  = Sprung mass of the vehicle

~~$\dot{z}_s$~~  = Sprung mass velocity in the

$\dot{z}_{sr}$  = Sprung mass velocity in the

$q_r$  = Pitch velocity of the sprung mass.

[0022] The wheel rotational dynamics shown in Figure 2 are given by the following equation:

$$\sum M_y = T_{bi} - F_{xi}R - F_{ri}R - T_{di} = -I_{wi} \dot{\omega}_i$$

$$\sum M_y = T_{bi} - F_{xi}R - F_{ri}R - T_{di} = -I_{wi} \dot{\omega}_i$$

where

$T_{bi}$  = Brake torque at i-th wheel

$\omega_i$  = Angular speed of i-th wheel

$F_{xi}$  = Longitudinal friction force at i-th tire contact patch

$R$  = Effective wheel rolling radius

$F_{ri}$  = Rolling Resistance at i-th tire contact patch

$T_{di}$  = Drive torque at i-th wheel

$I_{wi}$  = i-th wheel rotational inertia

$\dot{\omega}_i$  = Angular acceleration of i-th wheel.

[0023] For a braking event, the following set of equations of motion is written.

$$\frac{F_{xsumr} + F_{br} - F_{axr}}{M(\dot{V} - V_y r_r) + m_s \dot{z}_{sr}}$$

$$\frac{T_{bi} - F_{xi}R - F_{ri}R - T_{di}}{I_{wi}} = -\dot{\omega}_i$$

$$\frac{F_{xsumr} + F_{br} - F_{axr}}{M(\dot{V} - V_y r_r) + m_s \dot{z}_{sr} q_r}$$

$$\frac{T_{bi} - F_{xi}R - F_{ri}R - T_{di}}{I_{wi}} = -\dot{\omega}_i$$

(2)

u3  
conf

The pitch dynamics of the vehicle in the first equation is assumed to have negligible effect on the wheel braking forces. For the sake of simplicity, the effect of train forces arising out of road slopes and grades are also neglected. The drive torque (in a braking situation) is assumed to be insignificant in the second equation. Further simplification is made by assuming that the steer wheel angle is zero resulting in zero lateral motion. Also, the following relationships are defined:

$$F_{xi} = \mu_i F_{zi}; F_{ri} = \eta F_{zi}$$

where  $\mu_i(\kappa)$  = Friction Coefficient and  $\eta$  = Rolling Resistance Coefficient.

a4

**[0028]** The simplified equations of motion are then given by:

$$\begin{aligned} \sum \mu_i(\kappa_i) F_{zi} &= M\dot{V} \\ T_{bi} - \mu_i(\kappa_i) F_{zi} R &= -I_{wi} \dot{\omega}_i \end{aligned}$$

$$\begin{aligned} -\sum \mu_i(\kappa_i) F_{zi} &= M\dot{V} \\ T_{bi} - \mu_i(\kappa_i) F_{zi} R &= -I_{wi} \dot{\omega}_i \end{aligned}$$

a5

**[0031]** The sliding surface may be defined as follows,

$$\begin{aligned} \underline{\dot{S}} &= (\kappa_{th} - \kappa) \\ \underline{S} &= (\kappa_{th} - \kappa_l) \end{aligned} \quad (7)$$

a6

**[0032]** It is assumed here that the desired slip is the same as the slip threshold. With the above definition of the sliding surface, the sliding mode control law is given by,

$$\underline{\dot{S}} = -\eta \text{SAT}(\underline{\dot{S}})$$

$$\underline{\dot{S}} = -\eta \text{SAT}\left(\frac{\underline{S}}{\phi}\right)$$

where

$\eta$  = Convergence Factor;  $\phi$  = Boundary Layer Thickness

Further simplifying,

a6  
cont

$$\dot{\kappa}_{th} - \frac{R}{I_{wi}} \frac{1}{V} T_{bi} + \frac{R^2}{I_{wi}} \frac{1}{V} \alpha_{si} \kappa_i F_{zi} + \frac{R \omega_i}{V^2} \frac{1}{M} \sum \alpha_{si} \kappa_i F_{zi} = -\eta SAT\left(\frac{\kappa_{th} - \kappa_i}{\phi}\right) \quad (8)$$

Hence the control law is given by,

$$T_{bi} = \frac{VI_{wi}}{R} \dot{\kappa}_{th} + R \alpha_{si} \kappa_i F_{zi} + \frac{I_{wi}}{V} \frac{\omega_i}{M} \sum \alpha_{si} \kappa_i F_{zi} + \eta \frac{I_{wi}}{R} V * SAT\left(\frac{\kappa_{th} - \kappa_i}{\phi}\right) \quad (9)$$

a7

[0033] If  $\kappa_{th}$  is a constant, then the above control law becomes,

$$T_{bi} = R \alpha_{si} \kappa_i F_{zi} + \frac{I_{wi}}{V} \frac{\omega_i}{M} \sum \alpha_{si} \kappa_i F_{zi} + \eta \frac{I_{wi}}{R} V * SAT\left(\frac{\kappa_{th} - \kappa_i}{\phi}\right)$$

$$T_{bi} = R \alpha_{si} \kappa_i F_{zi} + \frac{I_{wi}}{V} \frac{\omega_i}{M} \sum \alpha_{si} \kappa_i F_{zi} + \eta \frac{I_{wi}}{R} V * SAT\left(\frac{\kappa_{th} - \kappa_i}{\phi}\right) \quad (10)$$

a8

[0034] Equation (10) is the proposed control law for the anti-lock braking system. As can be seen the brake torque (and the corresponding pressure) is dependent upon the normal force of the tire  $F_{zi}$  the tire slip and the value ~~chese~~ chosen for the peak slip angle.